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Perturbations of compact foliations II

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Introduction. A compact foliation F is one in which every leaf is compact. The problem we wish to consider concerns foliations F' whose plane fields are close, in some C^r-topology, to the plane field tangent to the leaves of F. Such an F' is called a C^r-perturbation. Then the following question arises: When does F' have a compact leaf ? The first result of this nature is due to H.Seifert [11]. He proved that any C^0-perturbation of the Hopf fibration $S^3 \longrightarrow S^2$ has a compact leaf. In the same paper, he showed that the theorem is also true for orientable S^1-bundles over surfaces B of $\chi(B) \neq 0$, where $\chi(B)$ is the euler characteristic number of B. The result was generalized by F.Fuller [5] to orientable circle bundles over arbitrary closed manifolds B with $\chi(B) \neq 0$. R.Langevin and H.Rosenberg [9] considered a fibration $p : E \longrightarrow B$ with fibre L, B a closed 2-manifold, E closed. They proved that any C^0-perturbation of this fibration has a compact leaf when $\pi_1(L)$ is isomorphic to Z, B is a sueface with $\chi(B) \neq 0$ and $\pi_1(B)$ acts trivially on $\pi_1(L)$. Furthermore the author [4] generalized the above result to compact codimension two foliations. The purpose of this note is to give the proofs of the results in [4] and generalize the above result to fibrations with B of general dimensions.

§1. Compact Hausdorff foliations and the generalized first return map.

Let M be a compact m-manifold without boundary and F a compact foliation of codimension q such that the leaf space M/F is Hausdorff. Such a foliation F is called a compact Hausdorff foliation. Then we have a nice picture of the local behavior of F as follows.

Proposition 1 (D.B.A.Epstein [3]). There is a generic leaf L_0 with property that there is an open dense subset of M, where the leaves have all trivial holonomy and are all diffeomorphic to L_0. Given a leaf L, we can describe a neighborhood $U(L)$ of L, together with the foliation on the neighborhood as follows. There is a finite group $G(L)$ of $O(q)$. $G(L)$ acts freely on L_0 on the right and $L_0/G(L) \cong L$. Let D^q be the unit q-disk. We foliate $L_0 \times D^q$ with leaves of the form $L_0 \times \{pt\}$. This foliation is preserved by the diagonal action of $G(L)$, defined by $g(x,y) = (x \cdot g^{-1}, g \cdot y)$ for $g \in G(L)$, $x \in L_0$ and $y \in D^q$. So we have a foliation induced on $U = L_0 \times_{G(L)} D^q$. The leaf corresponding to $y = 0 \in D^q$ is $L_0/G(L)$. Then there is a C^∞-imbedding $\varphi : U \longrightarrow M$ with $\varphi(U) = U(L)$, which preserves leaves and $\varphi(L_0/G(L)) = L$.

Definition 2. A leaf L is called singular if $G(L)$ is not trivial. The order of $G(L)$ is called the order of holonomy of L.

Definition 3. A singular leaf L is called isolated if the action of $G(L)$ has only the origin of D^q as fixed point.

From Proposition 1, we see that each isolated singular leaf is isolated, hence there are finitely many isolated singular leaves in F because of the compactness of M. Let S be the set of all non-isolated singular leaves of F. The leaf space M/F, which we denote by B, is a compact V-manifold of dimension q and the quotient map $\pi : M \longrightarrow B$ is a V-bundle(for definitions see I.Satake [10]).

Let $L_1,\dots,L_n$ denote all the isolated singular leaves of F with holonomy of order $k_1,\dots,k_n$ respectively. Put $p_i = \pi(L_i)(i=1,\dots,n)$ and $S_B = \pi(S)$. Note that $\pi: M - S\cup\{L_1,\dots,L_n\} \longrightarrow M - S\cup\{L_1,\dots,L_n\}/F \cong B - S_B\cup\{p_1,\dots,p_n\}$ is a natural fibration with generic leaf L as fibre. Thus $\pi_1(B - S_B\cup\{p_1,\dots,p_n\})$ acts on $\pi_1(L)$.

We assume that F satisfies the following conditions:

(C_1) $\pi_1(L) \cong Z$ for every leaf L of F,

(C_2) The codimension of each component of S in M is not equal to two.

Remark 4. From (C_1), $G(L)(L\in F)$ is isomorphic to a finite cyclic group. Hence S is a compact submanifold of M.

By Proposition 1, for each isolated singular leaf L_i, the restriction $\pi: \partial U(L_i) \cong L\underset{G(L_i)}{\times}\partial D^q \longrightarrow \partial D^q/G(L_i)$ is a fibration with compact fibre L and
$$\pi_1(\partial D^q/G(L_i)) = \begin{cases} G(L_i) & (q \geqq 3), \\ Z & (q = 2). \end{cases}$$
Thus we can see that $\pi_1(\partial D^q/G(L_i))$ acts trivially on $\pi_1(L)$ because that $G(L_i)$ is abelian. Furthermore we have $\pi_1(B - S_B) \cong \pi_1(B)$ from (C_2), hence we may consider that $\pi_1(B)$ also acts on $\pi_1(L)$ under the conditions (C_1) and (C_2).

Proposition 5. Let M be a compact manifold without boundary and F a compact Hausdorff foliation of codimension q with leaf space B, satisfying the conditions (C_1) and (C_2). Furthermore we assume that $\pi_1(B)$ acts trivially on $\pi_1(L)$. Let F' be a C^0-perturbation of F. Then there exists a vector field X on M satisfying the following;

i) X is orthogonal to F except for singular points of X and

ii) if there is a point $x\in M$ such that $X(x) = 0$, then the leaf

L' of F' through x is compact.

Proof. We shall associate to F' a diffeomorphism $f : M \longrightarrow M$ (a generalized first return map) as follows(see §1 of [9]). Fix a Riemannian metric on M so that F is a Riemannian foliation. Choose $\varepsilon > 0$ so that for each $x \in M$, the geodesics through x, of length ε, and orthogonal to $L(x)$, form a smoothly imbedded q-disk $D(x)$, where $L(x)$ is a leaf through x. We can suppose that for each leaf L, the disks $D(x)$, $x \in L$, form a tubular neighborhood $T(L)$ of L. Fix a generic leaf L and a point $x \in L$. Let α be a loop in L at x representing a generator of $\pi_1(L)$. Then for F' close to F, α can be lifted to a path on the leaf of F' through x, to a path starting at x and ending at a point of $D(x)$. This end point is denoted by $H(F', \alpha)$. H is the perturbed holonomy map(cf. M.W.Hirsch [6]). We define $f(x) = H(F', \alpha)(x)$. Now if y is another point of L, let β be any path in L from x to y(the length of β less than the diameter of L) and define $f(y) = H(F', \beta\cdot\alpha\cdot\beta^{-1})(y)$. This definition does not depend on β and defines a smooth map $f : L \longrightarrow M$.

Next we extend f to a map $f : T(L) \longrightarrow M$ by using the product structure in $T(L)$ and transporting α to each leaf in $T(L)$. Since $\pi : M - S \cup \{L_1, \ldots, L_n\} \longrightarrow B - S_B \cup \{p_1, \ldots, p_n\}$ is a fibration with fibre L and $\pi_1(B)$ acts trivially on $\pi_1(L)$, we can extend f to a map $f : M - S \cup \{L_1, \ldots, L_n\} \longrightarrow M$. Now we shall extend to each L_i ($i=1,\ldots, n$) and S. Let L be a generic leaf in $U(L_i)$(resp. $U(L_s)$, $L_s \in S$) and α a loop at x in L representing a generator of $\pi_1(L)$. We have a natural projection $j_i : L \longrightarrow L_i$ which is a k_i-fold covering ($i=1,\ldots,n$ and s; $k_s=2$). Let $\bar{\alpha}_i = j_i(\alpha)$ be a loop at $\bar{x} = j_i(x)$ in L_i. Then we define $f(\bar{x}) = H(F', \bar{\alpha}_i)(\bar{x})$ for $\bar{x} \in L_i$. This is well-defined since $\pi_1(L_i) \cong Z$. Then we see that the extended map $f : M \longrightarrow M$ is a smooth diffeomorphism. We can see that if $f(x)= x$ for some $x \in M$, then the leaf of F' through x is compact(see [8]). We associate to F a vector field X whose zero's give compact leaves. We have x and $f(x)$ in the geodesic disk $D(x)$ for $x \in M$. Let $X(x)$ be the vector tangent to the geodesic in $D(x)$ from to $f(x)$. Note that X is orthogonal to F. We easily see that if f has no fixed point, then X is never zero. This completes the proof.

§2. Statement of results for compact codimension two foliations.

Let F be a compact codimension two foliation with isolated singular leaves $L_1,\ldots,L_n$ of holonomy order $k_1,\ldots,k_n$ respectively. Furthermore we assume that $\pi_1(L) \cong Z$ for every leaf L of F. We let F' be a small perturbation of F. Then by the result of M.W.Hirsch([6], Theorem 1.1), we have the following: For each $U(L_i)$ $(i=1,\cdots,n)$, $F'|_{U(L_i)}$ has a compact leaf L_i' in $U(L_i)$ such that there is a diffeomorphism $h_i : L_i \longrightarrow L_i'$. We remark that F' has at least n compact leaves. Let α be a loop in a generic leaf L representing a generator of $\pi_1(L)$ and α_i(resp. $\alpha_i' = h_i(\alpha_i)$) a loop in L_i(resp. L_i') representing a generator of $\pi_1(L_i)$(resp. $\pi_1(L_i')$) such that $j_i(\alpha) = k_i \cdot \alpha_i$, where $j_i : L \longrightarrow L_i$ is the canonical projection. Let $H(\alpha_i)$ (resp. $H(\alpha_i')$) be the holonomy map of α_i(resp. α_i') for F(resp. F'), which is a local diffeomorphism of $(R^2,0)$. Thus if $H(\alpha_i)$ has no fixed point except for the origin 0, we can define the fixed point index of $H(\alpha_i)$ at 0 in the usual way. We denote it by $I(H(\alpha_i),L_i)$. Now we are in a position to state our main theorem.

Theorem 6. Let M be a compact manifold without boundary and F

a compact codimension two foliation of M with leaf space B. We assume that the fundamental groups of all leaves of F are isomorphic to Z and $\pi_1(B)$ acts trivially on $\pi_1(L)$. Let F' be a C^0-perturbation of F. If F' has exactly n isolated compact leaves, then we have a following relation;

$$\chi(B) + \sum_{i=1}^{n}\left(\frac{1}{k_i} - 1\right) = \sum_{i=1}^{n}\frac{1}{k_i} I(H(\alpha_i')^{k_i}, L_i').$$

The following corollary is an immediate consequence of Theorem 6 and this result is an extension of results of Seifert [11] and Langevin and Rosenberg [9].

Corollary 7. Let M be a compact manifold without boundary and F a compact codimension two foliation of M with leaf space B, which has no isolated singular leaves. Suppose that

1) $\pi_1(L) \cong Z$ for every leaf L of F,
2) $\pi_1(B)$ acts trivially on $\pi_1(L)$ and
3) $\chi(B) \neq 0$.

Then any C^0-perturbation of F has a compact leaf.

Example 8. The Klein bottle K^2 is an S^1-bundle over S^1 with structure group Z_2. Then we can construct a compact codimension one foliation G of K^2 such that G is transverse to the fibres and has two isolated singular leaves. We foliate $K^2 \times S^1$ with leaves of the form $L \times \{pt\}$, $L \in G$. This foliation F is a compact codimension two foliation with no isolated singular leaves and the leaf space $K^2 \times S^1/F$ is homeomorphic to a cylinder $S^1 \times [0,1]$. Thus the euler characteristic number of $K^2 \times S^1/F$ is equal to zero. Furthermore there exists a C^0-perturbation F' of F such that F' has no compact leaves. This example shows that the condition 3) of Corollary 7 is essential.

Corollary 9. Under the assumption of Theorem 6, we suppose that $H(\alpha_i')$ is expanding or contracting for $i=1,\ldots,n$. If $\chi(B) \neq n$, then F' has at least n+1 compact leaves.

Proof. We assume that F' has exactly n compact leaves. If $H(\alpha_i')$ is expanding or contracting, we have $I(H(\alpha_i'),L_i') = 1$. Thus from Theorem 6, we have $\chi(B) \neq n$, which contradicts the assumption.

Corollary 10. Under the assumption of Theorem 6, we suppose that 1 is not an eigenvalue of the linear holonomy $LH(\alpha_i')^{k_i} \in GL(2,R)$ for $i=1,\ldots,n$. If $\chi(B) < 0$, then F' has at least n+1 compact leaves.

Proof. We assume that F' has exactly n compact leaves. From the assumption, we can easily see that $I(H(\alpha_i')^{k_i},L_i')$ is equal to 1 or -1. Thus from Theorem 6, we have $\chi(B) \geq 0$, which is a contradiction.

Remark 11. Let M^3 be a closed manifold and F a foliation induced from a non-trivial S^1-action on M^3 with leaf space B. Then every leaf L of F is homeomorphic to S^1 and $\pi_1(B)$ acts trivially on $\pi_1(L)$.

Proof of Theorem 6. By the results of D.B.A.Epstein [2] and R.Edwards, K.Millett and D.Sullivan [1], every compact codimension two foliation is Hausdorff. S is a codimension one submanifold of M. Hence we can apply Proposition 5 for a small perturbation F' of F and the vector field X is defined. Note that S may be empty, but now we consider the case $S \neq \emptyset$. For simplicity, we assume that S is connected. It is proved similarly when S is not connected.

Let $U(L_i)$ be the saturated neighborhood of L_i (as in Proposition 2) and $V_1(S)$ the total space of the normal disk bundle of S in M.

Put $V_i = \pi(U(L_i))(i=1,\dots,n)$ which is a neighborhood of p_i and $V_0 = \pi(\nu_1(S))$ which is a neighborhood of $S_B=\partial B$. Let D be a 2-disk in B such that π is trivial over and $D\cap\{\bigcup_{i=0}^{n} V_i\} = \phi$. We identify D with the unit disk $D^2\subset R^2$ and $\pi^{-1}(D) = T \cong D\times L$. Let k be the least common multiple of $2,k_1,\dots,k_n$. Noting that $U(L_i)$ is the total space of the normal disk bundle of L_i in M, choose disjoint k/k_i disk fibres $W_i^j(j=1,\dots,k/k_i)$ for $i=1,\dots,n$. Then the restriction

$$\pi:\ \bigcup_{j=1}^{k/k_i}\partial W_i^j \longrightarrow \partial V_i \cong S^1 \text{ is a k-fold covering for } i=1,\dots,n.$$

Take a point $\bar{b}_i\in\partial V_i$ for each i. Put $\pi^{-1}(\bar{b}_i) = \{b_i^1,\dots,b_i^k\}$, $b_i^\ell \in W_i^{j(\ell)}$ for some $j(\ell)(1 \leq j(\ell) \leq k/k_i)$. On the other hand, the restriction $\pi: S\longrightarrow\partial B = S^1$ is a fibration. Thus we can construct k/2 disjoint sections of this fibration. We let these sections denote by T_1, $\dots,T_{k/2}$, which are identified with those images. The restricted bundle of $\nu_1(S)$ to T_i is denoted by $\mathbb{T}_i(i=1,\dots,k/2)$. Then the

$$\text{restriction } \pi:\ \bigcup_{i=1}^{k/2}\partial\mathbb{T}_i \longrightarrow \partial V_0 \cong S^1 \text{ is also a k-fold covering.}$$

Take a point $\bar{b}_0\in\partial V_0$. Put $\pi^{-1}(\bar{b}_0) = \{b_0^1,\dots,b_0^k\}$, $b_0\in \mathbb{T}_{r(\ell)}$ for some $r(\ell)(1 \leq r(\ell) \leq k/2)$. Since B is a compact topological 2-manifold with boundary, there is a cell complex K of $B - \bigcup_{i=0}^{n}\mathrm{int}(V_i)$ such that 1) D is contained in a 2-cell of K, 2) $\{\partial V_i\}$, $\{\bar{b}_i\}(i=0,1,\dots,k)$ are 1-cells, 0-cells of K respectively and 3) $B - \bigcup_{i=1}^{n}\mathrm{int}(V_i)-\mathrm{int}(D)$ is homotopy equivalent to $|K^{(1)}|$, where $|K^{(1)}|$ is the geometric realization of the 1-skeleton of K. Remark thet $\mathrm{int}(V_0)$ is homeomorphic to $\partial B\times[0,1)$. Then we can construct disjoint k sections

$s_1, \ldots, s_k$ over $|K^{(1)}|$ such that $s_\ell(\bar{b}_i) = b_i^\ell$ ($\ell=1,\ldots,k; i=0,1,\ldots,n$). In fact, for the case of dim $L \geq 2$, it is trivial. For the case of dim $L = 1$, that is, $L = S^1$, orienting $\{b_i^\ell\}$ ($\ell=1,\ldots,k; i=0,1,\ldots,n$) along the orientations of the fibres, we can construct disjoint k sections. Furthermore we extend these sections $s_1, \ldots, s_k$ to a tubular neighborhood N of $|K^{(1)}|$ in $B - \bigcup_{i=0}^{n} \mathrm{int}(V_i)$. We denote these sections by the same letters. Then we may assume, modifying $s_1, \ldots, s_k$ if necessary, that each $s_\ell(N)$ meets $W_i^{j(\ell)}$ and $T_{r(\ell)}$ along a segment in $\partial W_i^{j(\ell)}$ and $\partial T_{r(\ell)}$ respectively. Thus the union

$\bigcup_{i,\ell}\{s_\ell(N) \cup W_i^{j(\ell)} \cup T_{r(\ell)}\}$ is a compact 2-manifold, transverse to F

over $N \cup \{\bigcup_{i=0}^{n} V_i\}$. Since B - int(D) is homotopy equivalent to

$N \cup \{\bigcup_{i=0}^{n} V_i\}$, we have the following proposition.

Proposition 12. There exists a compact connected 2-manifold B^*, transverse to F over B - int(D) such that 1) $\pi : B^* \longrightarrow B - \mathrm{int}(D)$ is a k-fold covering except for $p_1, \ldots, p_n$ and ∂B(if $\partial B \neq \emptyset$) and 2) B^* meets ∂T in simple closed curves C_i($i=1,\ldots,r$), where r is a divisor of k.

The vector field X projects naturally a vector field X^* tangent to B^* since X and B^* are transverse to F. If F' has exactly n compact leaves, X^* has $k\sum_{i=1}^{n} 1/k_i$ isolated singular points in $B^* \cap (\bigcup_{i=1}^{n} U(L_i))$ and X^* is never zero outside of $B^* \cap (\bigcup_{i=1}^{n} U(L_i))$. Note that the sum of indices of singular points of X^* is equal to

$k\sum_{i=1}^{n} \frac{1}{k_i} I(H(\alpha_i')^{k_i}, L_i')$ since $H(F', \alpha_i) = H(\alpha_i')$ on $D(\bar{x})(\bar{x} \in L_i)$.

Let $L_0 = \pi^{-1}(0)$, $0 \in D$ and α be a loop in L_0 representing a generator of $\pi_1(L_0) \cong Z$. R.Langevin and H.Rosenberg have proved the following proposition which is essentially due to Seifert [11].

Proposition 13. $I(F',\alpha) = 0$ (for definition and proof, see [9]).

We construct a closed 2-manifold $\bar{E}^*$ pasting r disks D^2_ℓ ($\ell=1,\dots,r$) to B^* along $\partial B^* = \bigcup_{\ell=1}^{r} C_\ell$: $\bar{B}^* = B^* \cup D^2_1 \cup \dots \cup D^2_r$. The vector field $\pi_*(X(C_i))$ on ∂D is homotopic to the vector field $\pi_*(X(n_i\alpha))$, where n_i is the integer defined by $[C_i] = n_i[\alpha]$ and $[\ ]$ denotes homotopy class in $\pi_1(T)$. From Proposition 13, $\pi_*(X(\alpha))$ is homotopic to the constant vector field, hence deforming X^* along ∂B^* in the homotopy class, we may assume that $\pi_*(X^*|_{\partial B^*})$ is a constant vector field on ∂D. Since π: $C_i \longrightarrow \partial D \cong S^1$ is a k/r-fold covering, we easily see that the vector field X^* is extended to a vector field $\bar{X}$ on $\bar{B}^*$ with exactly r singular points of index $-(\frac{k}{r}-1)$ in $\bigcup_{\ell=1}^{r} D^2_\ell$.

Now the euler characteristic number of $\bar{B}^*$ is equal to

$\chi(\bar{B}^*) = -r(\frac{k}{r}-1) + k\sum_{i=1}^{n} \frac{1}{k_i} I(H(\alpha_i^!)^{k_i}, L_i^!)$. On the other hand,

$\chi(\bar{B}^*) = \chi(B^* \cup D^2_1 \cup \dots \cup D^2_r) = k\left\{\chi(B) + \sum_{i=1}^{n}(\frac{1}{k_i}-1) -1\right\} + r$. Hence we have the required relation.

§4. Remarks.

Theorem 14. Let π: $M \longrightarrow B$ be a fibration with B a closed oriented k-manifold and compact fibre L. Suppose that 1) $\pi_1(L) \cong Z$, 2) $\pi_1(B)$ acts trivially on $\pi_1(L)$, 3) π^*: $H^k(B;Z) \longrightarrow H^k(M;Z)$ is injective and 4) $\chi(B) \neq 0$. Then any C^0-perturbation of the foliation induced from this fibration has a compact leaf.

Proof. Let F' be a small perturbation of F. We assume that F' has no compact leaves. Applying Proposition 5 for such an F', there exists a vector field X on M such that X is orthogonal to F. Let $\chi \in H^k(B;Z)$ be the euler class of T(B). The image $\pi^*\chi \in H^k(M;Z)$ is the primary obstruction of a section of the induced bundle $\pi^*T(B)$ over M. $\pi^*T(B)$ has the non-zero section X, so $\pi^*\chi = 0$. From the condition 3), $\chi = 0$, hence $\chi(B) = 0$ which contradicts the condition 4).

Now we shall apply Main theorem of Hirsch and Thurston [7] to Theorem 14. First we prepare the following terminology(see [7]).

Definition 15. A smooth bundle $\xi = (\pi, M, B)$ is called a foliated bundle if ξ has a foliation whose leaves are transverse to the fibres and of complementary dimension.

As well-known, a smooth bundle ξ with compact fibre L is a foliated bundle with holonomy group Γ if and only if ξ is a (Γ, L)-bundle with discrete structure group Γ.

Definition 16. A group Γ is called amenable if there exists a left-invariant linear function A on $B(\Gamma)$, the bounded functions on Γ, such that $A(f) \geq 0$ when $f \geq 0$ and $A(1) = 1$.

Remark 17. The class of amenable groups is closed under the operations of taking quotients, subgroups, extensions of amenable groups by amenable groups and direct limits; and contains all abelian groups and finite groups.

Notation 18. We denote by $\mathcal{C}$ the smallest class of groups that contains all amenable groups and is closed under finite extensions and free products.

Remark 19. $\Gamma \in \mathcal{C}$ if Γ is solvable, free or of subexponential growth(see [7]).

Definition 20. A content on a space V is by definition a linear functional S on the space of continuous real functions on V such that $S(c) = c$ for any constant function c.

Theorem 21. Let $\xi = (\pi, M, B)$ be a foliated bundle with B a closed k-manifold compact fibre L and holonomy group Γ.

Suppose that 1) $\pi_1(L) \cong Z$,

2) $\pi_1(B)$ acts trivially on $\pi_1(L)$,

3) (a) Γ preserves a content on L or (b) $\Gamma \in \mathcal{C}$ and

4) $\chi(B) \neq 0$.

Then any C^0-perturbation of ξ has a compact leaf.

Proof. Passing to a double covering of B, if necessary, we may assume that B is orientable. From the condition 3) we have Main theorem of Hirsch and Thurston [7]. Hence it follows that $\pi^* : H^k(B;R) \longrightarrow H^k(M;R)$ is injective. Since B is oriented, $H^k(B;Z) \cong Z$, therefore $\pi^* : H^k(B;Z) \longrightarrow H^k(M;Z)$ is injevtive. Hence Theorem 21 is a corollary to Theorem 14.

Remark 22. $\pi_1(B) \in \mathcal{C}$ implies $\Gamma \in \mathcal{C}$, for Γ is the image of the holonomy homomorphism $\pi_1(B) \longrightarrow \mathrm{Diff}(L)$ of the foliated bundle ξ.

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